JACOBS UNIVERSITY

Mathematics

Bachelor's Degree Program (BSc)

Mathematics

Program Handbook Bachelor of Science

Academic Year 2012–13

Contents

1 Mathematics at Jacobs University

Mathematics encompasses a broad range of topics, ranging from the beauty and satisfaction of pure thought (in areas such as algebra, complex analysis, geometry or topology) to the usefulness of practical applications which is no less satisfying (for example, when improving signal and image processing using wavelets, or modeling fluid using partial differential equations). Often enough, the purest of mathematics finds intriguing practical applications in surprising ways (number theory is used heavily in cryptography, wavelets and dynamical systems are at work in engineering, mathematical game theory has won a Nobel prize in economics, etc).

1.1 The Mathematics Curriculum

The mathematics curriculum at Jacobs University has been designed with a number of key features in mind:

- Flexibility: strong students can move ahead at *their* pace
- Choice: cater for students with different interests and backgrounds
- Solidity: assure that students master the core knowledge expected of any mathematics graduate
- Compatibility: with strong international graduate programs
- Challenge: for strong students
- Coherence: courses are designed by content and interdependence so as to form a coherent education, not as isolated units
- Breadth: graduates have acquired an overview of mathematical areas and perspectives beyond core courses
- Transdisciplinarity: students take electives throughout the school of Engineering and Science, HSS courses and University Studies Courses

Depending on their previous educational system, age, and background, strong incoming mathematics students can have very different levels of preparation. Therefore, we offer at least two main entry-points into our undergraduate program, with a number of possible intermediate and even stronger variants.

The *regular variant* of the curriculum is suitable for entering students without a lot of training in formal mathematics in high school and for students who are undecided between several majors. Regular variant students will begin their their study taking GENERAL MATHEMATICS AND COMPUTATIONAL SCIENCE. This module provides an introduction to key concepts such as formal reasoning and proofs, and an overview of important areas in mathematics. It is usually taken together with introductory courses in two other subjects, and students can choose at the end of the first year which major to pursue.

The *advanced variant* of our curriculum is designed for particularly well-prepared students who enter the program often with experience of mathematical olympiads and competitions or with a specialized mathematics education at their high schools. These students begin with courses in ANALYSIS and LINEAR ALGEBRA (formally comparable to introductory classes at German universities, or second/third-year classes at North American universities, but at a relatively demanding level). Students following the advanced variant of the curriculum typically take specialized courses in their second year, and a mix of specialized and graduate level courses in their third year.

We offer our students individual advising between the various variants our curriculum offers, and students are generally very happy about the individualized possibilities. Some students, upon personal advice, have taken even more advanced classes in their first years. In other words, to attract the strongest students in mathematics, Jacobs University offers an education which picks them up where they are, and which offers challenges at all levels. These challenges may consist of particularly strong or advanced classes, of particularly interesting problems within regular classes, of competitions and olympiads on campus or at an international level, or in contact early on with the research groups of the faculty, including work on open research problems.

A particular feature of the mathematics undergraduate curriculum is that students should obtain a broad fundamental education in mathematics which gives them an overview of mathematics, including directions of current mathematical research in several areas and relations between various subjects, rather than specialized education in particular areas of mathematics. Consequently, undergraduate courses are broad in content and emphasize cross-relations, rather than as sequences of ever-more specialized classes. In addition, the courses PERSPECTIVES OF MATHEMATICS offer links and visions on areas not usually found in general introductory classes, and students are encouraged to engage in supervised research from early on.

The requirements for all variants of our curriculum ensure that graduates are well prepared for continuing their studies in mathematics graduate programs at leading universities worldwide. At the very minimum, graduates will be ready for the standard beginning mathematics courses for graduate programs at leading North American universities (graduate level Real and Complex Analysis, Algebra, Topology); similarly, in the French system, students will be prepared to enroll in DEA programs.

Many of our former undergraduates who went to these leading programs reported that they were better prepared by their education at Jacobs than their fellow graduate students. Others have joined industry or business and started successful non-academic careers right after their Bachelor degrees.

1.2 Study abroad

While the mathematics program is self-contained, we encourage our students to spend a semester or a year abroad, and students have accepted these options (Paris/Orsay, Rice, etc). Through personal advising and our flexible curriculum we ensure on-time graduation of students who study abroad. Personal faculty contacts with a number of institutions ensure that semester abroad students find a good education abroad.

1.3 Career Options

The mathematics curriculum at Jacobs University is designed to prepare students for work towards a Ph.D. in the strongest graduate programs worldwide: a head start into an international career in academic research. At the same time, graduates have acquired skills such as abstract reasoning, logical thinking and endurance which are well sought after by non-academic employers. Consequently, mathematicians enjoy a large and growing choice of top jobs even outside of the university world, for example in research and development, finance, banking, and management. All our program alumni have found rewarding and interesting occupations in a large variety of options.

2 Modules

Where appropriate, several closely interconnected courses with an overarching set of learning goals are grouped into a *module*. Modules provide a higher level view into the curriculum structure to facilitate information and documentation. Note, however, that formal graduation requirements and the documentation of the learning process refer to credit points and grades attributed to the individual courses or lab units.

ESM for Mathematics Majors

Semester: 1–2
Credit Points: 10

General Information Students of Mathematics may either take Engineering and Science Mathematics courses ("regular variant" of the curriculum) or ANALYSIS and LINEAR ALGEBRA ("advanced variant" of the curriculum) during their first year of study. Following the advanced variant is recommended; students should consult with Mathematics faculty for advice.

Learning goals

- Basic skills in differential and integral calculus, linear algebra, probability, and statistics
- Problem solving skills
- Training in abstract reasoning and symbolic manipulation
- Ability to turn real-world problems into a concise mathematical question
- Ability to interpret mathematical statements back into the problem domain

Courses Take at least one course per semester from among the following.

120101 ESM 1A – Single Variable Calculus

120111 ESM 1B – Multivariable Calculus, ODE

120102 ESM 2A – Linear Algebra, Probability, Statistics

120112 ESM 2B – Linear Algebra, Fourier, Probability

General Mathematics and Computational Science

Semester: 1–2
Credit Points: 15

General Information This module is taken by all students who follow the regular track of the Mathematics and ACM Bachelor degrees. Students with a strong background in Mathematics should take ANALYSIS and LINEAR ALGEBRA in their first year.

Learning goals

- Fluency in basic logic
- Ability to construct rigorous and concise arguments
- Problem solving skills
- Confidence in formulating conjectures
- Ability to turn real-world problems into a concise mathematical question
- Ability to interpret mathematical statements back into the problem domain
- Ability to build simple models that still capture the essence of a problem
- Basic knowledge in using standard software tools

Courses

110101 General Mathematics and Computational Science I

110111 Symbolic Software Lab

110102 General Mathematics and Computational Science II

110112 Numerical Software Lab

ANALYSIS

Semester: 3–4 (regular variant) or 1–2 (advanced variant)
Credit Points: 15

General Information This module provides the foundations in Mathematical Analysis for students of Mathematics and ACM. In addition, it may be an appropriate first year or second year choice for students particularly in Physics, Electrical Engineering and Computer Science. Consult with faculty from your major, and read the document "How to choose Mathematics courses".

The courses in this module have no formal prerequisites; incoming students with a strong mathematics background are encouraged to take this module in their first year of study ("Advanced Track" of the Mathematics curriculum). However, a familiarity with mathematical reasoning and proof (e.g. proof by induction or by contradiction) is assumed. The module GENERAL MATHEMATICS AND COMPUTATIONAL SCIENCE is designed for students who still need to develop this maturity.

Learning goals

- Core skills in Analysis that every Mathematician needs
- Working knowledge on series, differentiation and integration on R^n
- Develop these concepts in a rigorous manner and in sufficient generality to prepare the student for advanced work in mathematics.
- Building a core set of examples and counterexamples for each concept area
- Necessary skills needed for more advanced and specialized courses

Courses

100211 Analysis I

100212 Analysis II

LINEAR ALGEBRA

Semester: 3–4 (regular variant) or 1–2 (advanced variant)
Credit Points: 15

General Information This module complements ANALYSIS in the fundamental education of the Mathematics major. Students of ACM should consider taking this module. It may also be an appropriate first year or second year choice for students particularly in Physics, Electrical Engineering and Computer Science. Consult with faculty from your major, and read the document "How to choose Mathematics courses".

The courses in this module have no formal prerequisites; incoming students with a strong mathematics background are encouraged to take this module in their first year of study ("Advanced Track" of the Mathematics curriculum). However, a familiarity with mathematical reasoning and proof (e.g. proof by induction or by contradiction), is assumed. The module GENERAL MATHEMATICS AND COMPUTATIONAL SCIENCE is designed for students who still need to develop this maturity.

Learning goals

- Core skills in Linear Algebra that every mathematician needs
- Ability to abstract linear structures in any application domain to studying linear maps on vector spaces
- Ability to carry out concrete computations in this framework
- Working knowledge of the fundamental algebraic structures, in preparation of the respective third year courses

Courses

100221 Linear Algebra I

100222 Linear Algebra II

Perspectives of Mathematics

Semester: 3–4 (regular variant) or 1–2 (advanced variant)
Credit Points: 5–10

General Information While the mathematical core courses build the foundation for a solid mathematical knowledge, the courses in this module cover beautiful or interesting areas of mathematics which need not be part of a systematic education, but which show mathematics as a lively, active and varied subject, and which convey the spirit why mathematicians enjoy their field. The topics covered in each instance of a course vary, so that each course may be taken for credit more than once.

Learning goals

- Ability to apply basic skills to larger, but well defined mathematical problems
- Find and work with original literature
- Mathematical writing and presentation skills

Courses

100291 Perspectives of Mathematics I

100292 Perspectives of Mathematics II

Specialized Mathematics Courses

Semester: 5–6 (regular variant) or 3–6 (advanced variant)
Credit Points: varying

General Information Third year courses in Mathematics are designed to give students a first in-depth look into a wide selection of specialization areas in Mathematics. They also refine and extend the fundamental concepts introduced in Analysis and Linear Algebra. The courses are designed to be as independent as possible, so that a flexible selection of courses is possible. Some spring semester courses will depend to a limited extent on fall semester courses, and students should consult the individual course descriptions for details.

Learning goals

- Skills in various areas of Mathematics as are required for a successful entry to graduate programs in Mathematics, Applied Mathematics, Computational Science, and Financial Mathematics
- Sound basis on which students can develop their further career goals
- Experience with advanced concepts needed by research mathematicians
- First contact with the various specialization areas in Mathematics

A detailed list of available courses is provided in Section 5 below.

Guided Research Mathematics and BSc Thesis

Semester: 5–6
Credit Points: 15

General Information In this module, students work on a research project in a particular area of specialization within mathematics. A faculty member acts as a supervisor and works with the student in a small study group or on a one-on-one basis.

Guided research has three major components: Literature study, research project, and seminar presentation (including a written report). The Guided Research report in the spring semester will typically be the Bachelor's Thesis which is a graduation requirement for Jacobs University undergraduates.

Learning goals

- Learn how to search and use research literature
- Scientific writing skills
- Ability to orally present a project and its results
- Time management and organizational skills
- First-hand research experience

Courses

100391 Guided Research Mathematics I

100392 Guided Research Mathematics II

3 Requirements for the Bachelor of Science Degree

3.1 General Requirements

To obtain a B.Sc. degree at Jacobs University a minimum of 180 ECTS credit points must be earned over a period of 6 semesters.

- A minimum of 140 ECTS credits must be earned in the School of Engineering and Science.
- 30 ECTS credits must be earned through transdisciplinary courses, comprised of courses in the School of Humanities and Social Sciences (SHSS) and University Study Courses (USC). The choice between SHSS courses and USCs is free.
- Up to 4 language courses (up to 10 ECTS credit points) may be counted toward Home School Electives.
- All undergraduate students are required to complete an internship, normally to be accomplished between the second and third year of study. Information about the internship will be listed on the transcript. The internship must last at least two consecutive months. No credits are connected to the internship requirement.
- It is mandatory to successfully complete a Bachelor Thesis in Mathematics. This thesis needs to be supervised by one or several faculty memebers, at least one from Mathematics. Writing the thesis is formally part of *Guided Research Mathematics and BSc Thesis II*. Usually, *Guided Research Mathematics I* serves as a preparation to write the thesis.

3.2 Requirements of the Major

Course requirements: Students must obtain at least

- 25 ECTS credits for Mathematics courses at year 1 level or above.
- 40 ECTS credits for Mathematics courses at year 2 level or above.
- 45 ECTS credits for Mathematics courses at year 3 level or above.

The core courses Analysis I/II and Linear Algebra I/II have to be completed successfully to graduate.

Note: The following classes qualify as *Mathematics courses* in the requirements above: Courses listed as Mathematics (course numbers 100xyz), Applied and Computational Mathematics 110xyz), and Mathematics Service (course numbers 120xyz). Here x denotes the year of the course. For example 100312 Introduction to Complex Analysis is a year 3 level course. The course recommendations below explain the more advanced options of the curriculum.

4 Recommended Course Plans

In Mathematics, there are two main variants of the curriculum. In the regular variant, students will start out taking Engineering and Science Mathematics and General Mathematics and Computational Science. Students following the regular variant will satisfy all degree requirements. Students with a strong Mathematics background will want to take the advanced variant which starts out with Analysis and Linear Algebra and moves to more advanced courses from there.

Intermediate or even more advanced variants are equally possible, subject only to the formal graduation requirements; students should generally try to take as demanding courses as possible.

4.1 Regular Variant

Year 1 Courses	**Fall**	**C**	**T**	**Spring**	**C**	**T**
ESM for Mathematics	120111	*5*	m	120112	*5*	m
General Mathematics & CPS	110101	*5*	m	110102	*5*	m
incl. Natural Science Lab Units	110111	*2.5*	m	110112	*2.5*	m
First year courses in ESc subject		*5*	e		*5*	e
Incl. Natural Science Lab Units		*2.5*	e		*2.5*	e
First year courses in second ESc subject [1]		*5*	e		*5*	e
Transdisciplinary Courses		*5*	u		*5*	u
Running Total / *Semester Total*	30	*30*		60	*30*	
Year 2 Courses	**Fall**	**C**	**T**	**Spring**	**C**	**T**
Analysis	100211	*7.5*	m	100212	*7.5*	m
Linear Algebra	100221	*7.5*	m	100222	*7.5*	m
Perspectives of Mathematics	100291	*5*	m	100292	*5*	m
Language Courses or Home School Electives		*5*	e		*5*	e
Transdisciplinary Courses		*5*	u		*5*	u
Running Total / *Semester Total*	90	*30*		120	*30*	
Year 3 Courses	**Fall**	**C**	**T**	**Spring**	**C**	**T**
Specialized Mathematics Courses		*22.5*	m		*15*	m
Guided Research/BSc Thesis Mathematics				100392	*7.5*	m
Language Courses or Home School Electives		*2.5*	e		*2.5*	e
Transdisciplinary Courses		*5*	u		*5*	u
Running Total / *Semester Total*	150	*30*		180	*30*	

C = ECTS credit points, T=type (m=mandatory, e=elective, u=university), Transdisciplinary Courses are School of Humanities and Social Sciences and University Studies Courses

4.2 Advanced Variant

Note: The course plan of the advanced variant exceeds the formal graduation requirements. By taking a moderate number of credits in excess of the required minimum, students can combine an excellent preparation for graduate study in Mathematics while still developing interests outside of the Mathematics major.

Year 1 Courses	**Fall**	**C**	**T**	**Spring**	**C**	**T**
Analysis	100211	*7.5*	m	100212	*7.5*	m
Linear Algebra	100221	*7.5*	m	100222	*7.5*	m
Perspectives of Mathematics	100291	*5*	m	100292	*5*	m
Mathematics Lab Units	110111	*2.5*	m	110112	*2.5*	m
Courses in ESc subject [1]		*5*	e		*5*	e
Associated Natural Science Lab Units		*2.5*	e		*2.5*	e
Transdisciplinary Courses		*5*	u		*5*	u
Running Total / *Semester Total*	35	*35*		70	*35*	

Year 2 Courses	**Fall**	**C**	**T**	**Spring**	**C**	**T**
Specialized Mathematics Courses		*22.5*	m		*22.5*	m
Language Courses or Home School Electives		*5*	e		*5*	e
Transdisciplinary Courses		*5*	u		*5*	u
Running Total / *Semester Total*	102.5	*32.5*		135	*32.5*	

Year 3 Courses	**Fall**	**C**	**T**	**Spring**	**C**	**T**
Specialized/Advanced Courses		*15*	m		*15*	m
Guided Research/BSc Thesis Mathematics	100391	*7.5*	m	100392	*7.5*	m
Language Courses or Home School Electives		*5*	e		*5*	e
Transdisciplinary Courses		*5*	u		*5*	u
Running Total / *Semester Total*	167.5	*32.5*		200	*32.5*	

C = ECTS credit points, T=type (m=major, e=elective, u=university), Transdisciplinary Courses are School of Humanities and Social Sciences and University Studies Courses

4.3 Notes

1. While language courses are optional and can be taken any time during the course of studies, students are advised to begin taking language courses early in addition to regular Engineering and Science electives.

4.4 Recommendation Professional Skills

The SES recommends attending the Professional Skills seminars offered by the Career Services Center. Those seminars include soft skills development seminars and application training which will help you to cope with your studies and master your internship and job search.

For more information on internships see http://www.jacobs-university.de/career-services/internship.

5 Courses

5.1 First year Mathematics and ACM

110101: General Mathematics and Computational Science I

Short Name: GenMathCPS I
Type: Lecture
Credit Points: 5
Prerequisites: None
Corequisites: None
Tutorial: Yes

Course contents General Mathematics and Computational Science I and II are the introductory first year courses for students in *Mathematics* and *Applied and Computational Mathematics*. In addition, these courses address anyone with an interest in mathematics and mathematical modeling. Each semester includes a selection of "pure" and "applied" topics which provide a solid foundation for further study, convey the pleasure of doing mathematics, and relate mathematical concepts to real-world applications.

Topics covered in the first semester are:

- *Fundamental concepts:* sets, relations, functions, equivalence classes.
- *Numbers:* Peano axioms, proof by induction, construction of integers and rational numbers.
- *Discrete Mathematics:* combinatorics, binomial coefficients, generating functions, applications to elementary discrete probability.
- *Inequalities:* Geometric-arithmetic mean inequalities, Cauchy inequality; Laplace's method and Stirling's approximation.
- *Difference equations:* linear first order difference equations, nonlinear first order difference equations, equilibrium points and their stability, linear second order difference equations; modeling with difference equations.

110102: General Mathematics and Computational Science II

Short Name: GenMathCPS II
Type: Lecture
Credit Points: 5
Prerequisites: 110101
Corequisites: None
Tutorial: Yes

Course contents This course continues *General Mathematics and Computational Science I* with the following selection of topics:

- *Groups:* Basic properties and simple examples, Euclidean symmetries of the plane, symmetry groups of subsets of the plane, symmetry groups of polyhedra.

- *Graph Theory:* Graphs and parity, trees, Euler's formula and Euler characteristic, pairings, Eulerian graphs.
- *Stochastic Modeling:* Simple discrete stochastic systems, continuum limits, introduction to entropy.
- *Linear Programming:* graphical method, simplex method, duality.
- *Fourier Transform:* Discrete Fourier transform, fast Fourier transform, Fourier transform on groups.

110111: Natural Science Lab Unit – Symbolic Software

Short Name:	NatSciLab SymbSoft
Type:	Lab
Credit Points:	2.5
Prerequisites:	None
Corequisites:	None
Tutorial:	No

Course contents The Natural Science Lab Units in Mathematics and ACM will introduce the computer as a tool for the working mathematician, as well as for scientists in many other fields.

The Lab Unit *Symbolic Software* introduces *Mathematica*, a software package that can perform complex symbolic manipulations such as solving algebraic equations, finding integrals in closed form, or factoring mathematical expressions. Mathematica also has powerful and flexible graphing capabilities that are useful for illustrating concepts as well as numerical data. The computer will be used as a tool in this course so that you will also learn some mathematics alongside learning to use the computer program.

110112: Natural Science Lab Unit – Numerical Software

Short Name:	NatSciLab NumSoft
Type:	Lab
Credit Points:	2.5
Prerequisites:	None
Corequisites:	None
Tutorial:	No

Course contents The Natural Science Lab Units in Mathematics and ACM will introduce the computer as a tool for the working mathematician, as well as for scientists in many other fields.

The Lab Unit *Numerical Software* introduces *Matlab* and its free cousin *Octave*, software packages that allow easy and in many cases efficient implementations of matrix-based "number crunching". The software is ideal for numerical work such as solving differential equations or analyzing large amounts of laboratory data. The computer will be used as a tool in this course so that you will also learn some mathematics alongside learning to use the computer program.

This Lab Unit is particularly suited for students from both schools interested in experiments, as *Matlab* is used as a standard tool for analyzing and visualizing data in many fields of research.

5.2 Engineering and Science Mathematics

120101: ESM 1A – Single Variable Calculus

Short Name:	ESM 1A
Type:	Lecture
Credit Points:	5
Prerequisites:	None
Corequisites:	None
Tutorial:	Yes

Course contents The courses from the Engineering and Science Mathematics 1 series provide the foundation for all other Engineering and Science Mathematics courses. Taking at least one of them is mandatory for all Engineering and Science majors. Emphasis is on the use of basic mathematical concepts and methods in the sciences, rather than on detailed proofs of the underlying mathematical theory.

The course ESM 1A covers basic differential and integral calculus of functions of one variable. It starts with a brief review of number systems and elementary functions, then introduces limits (for both sequences and functions) and continuity, and finally derivatives and differentiation with applications (tangent problem, error propagation, minima/maxima, zero-finding, curve sketching). A short chapter introduces complex numbers.

The second half of the semester is devoted to integration (anti-derivatives and Riemann integral) with applications, and concluded by brief introductions to scalar separable and linear first-order differential equations, and the convergence of sequences and power series.

Compared to ESM 1C which covers similar material, this course assumes a rigorous high school preparation in Mathematics and leaves more room for explaining mathematical concepts (as needed for the majority of SES majors).

120111: ESM 1B – Multivariable Calculus, ODE

Short Name:	ESM 1B
Type:	Lecture
Credit Points:	5
Prerequisites:	None
Corequisites:	120101
Tutorial:	Yes

Course contents Engineering and Science Mathematics 1B introduces multivariable calculus and ordinary differential equations, topics of particular importance to the physical sciences. Students of ACM, Physics, and Electrical Engineering are strongly encouraged to take this course in their first semester. The curriculum is designed so that ESM 1A and ESM 1B can be taken at the same time.

The course covers vector algebra (three-dimensional vectors, dot product, cross product), equations of lines, planes, and spheres, Euclidean distance, vector-valued functions, space curves, functions of several variables, partial derivatives, chain rule, gradient, directional derivative, extrema, Lagrange multipliers, double and triple integrals with applications, change of variables, vector fields, divergence, curl, cylindrical and spherical coordinates, line integrals, Green's theorem in the plane, surface and volume integrals, divergence theorem, Stokes' theorem, introduction to ordinary differential equations (direction field, the question of existence and uniqueness of solutions), separable and exact equations, integrating factors, and linear higher order ODEs with constant coefficients.

120102: ESM 2A – Linear Algebra, Probability, Statistics

Short Name:	ESM 2A
Type:	Lecture
Credit Points:	5
Prerequisites:	None
Corequisites:	None
Tutorial:	Yes

Course contents Second semester Engineering and Science Mathematics is offered in two parallel classes that cover a common set of core topics at approximately the same level of difficulty. However, style of exposition and selection of additional material will vary slightly to meet the needs of different groups of majors.

ESM 2A is recommended for students majoring in Life Sciences or Chemistry. It covers the following topics: Linear Algebra (equations of lines and planes, matrix algebra, system of linear equations, matrix inverse, vector spaces, linear independence, basis, dimension, linear transformations, change of basis, eigenvalues and eigenvectors, diagonalization). Probability (basic notions of set theory, outcomes, events, sample space, probability, conditional probability, Bayes' rule, permutations and combinations, random variables, expected value, variance, binomial, Poisson, and normal distributions, central limit theorem). Statistics (one-sample hypothesis testing, two sample hypothesis testing, chi-square hypothesis testing, analysis of variance, bivariate association, simple linear regression, multiple regression and correlation).

120112: ESM 2B – Linear Algebra, Fourier, Probability

Short Name:	ESM 2B
Type:	Lecture
Credit Points:	5
Prerequisites:	120101 or 120111 or 120121
Corequisites:	None
Tutorial:	Yes

Course contents Second semester Engineering and Science Mathematics is offered in two parallel classes that cover a common set of core topics at approximately the same level of difficulty. However, style of exposition and selection of additional material will vary slightly to meet the needs of different groups of majors.

ESM 2B is recommended for students who do not intend to major in the Life Sciences or Chemistry. It covers the following topics:

- Linear Algebra (equations of lines and planes, matrix algebra, system of linear equations, matrix inverse, vector spaces, linear independence, basis, dimension, linear transformations, change of basis, eigenvalues and eigenvectors, diagonalization, inner products, orthonormalization)

- Fourier methods (expanding functions in terms of orthonormal function systems, Fourier series, Fourier transform, Dirac delta-function)
- Probability (basic notions of set theory, outcomes, events, sample space, probability, conditional probability, Bayes' rule, permutations and combinations, random variables, expected value, variance, binomial, Poisson, and normal distributions, central limit theorem).

120202: ESM 4A – Numerical Methods

Short Name:	ESM 4A
Type:	Lecture
Credit Points:	5
Prerequisites:	120112 or 100221
Corequisites:	None
Tutorial:	No

Course contents Engineering and Science Mathematics 4A is mandatory for students of Electrical Engineering, Computer Science, and Applied and Computational Mathematics. It is also recommended as a home school elective for students who would like to get a short, one-semester introduction to Numerical Methods.

This course is a hands-on introduction to numerical methods. It covers root finding methods, solving systems of linear equations, interpolation, numerical quadrature, solving ordinary differential equations, the fast Fourier transform, and optimization. These methods are crucial for anyone who wishes to apply mathematics to the real world, i.e. computer scientists, electrical engineers, physicists and, of course, mathematicians themselves.

5.3 Second year Mathematics

100211: Analysis I

Short Name:	Analysis I
Type:	Lecture
Credit Points:	7.5
Prerequisites:	None
Corequisites:	None
Tutorial:	Yes

Course contents Analysis I/II is one of the fundamental courses in the mathematical education (together with Linear Algebra I/II). Its goal is to develop calculus in a rigorous manner and in sufficient generality to prepare the student for advanced work in mathematics. At the same time, the content is chosen so that students arrive quickly at central concepts which are used in essentially all mathematics courses, and which are needed in the exact sciences.

The Analysis sequence begins with a quick review of natural, rational and real numbers (which are assumed as known), and introduces the field of complex numbers. The axiom of completeness distinguishes the real numbers from the rationals and marks the beginning of Analysis. The complex exponential and trigonometric functions are defined.

Metric spaces are introduced and used to define continuity and convergence in a general framework. The Bolzano-Weierstraß and the Heine-Borel theorems are proved. The intermediate and maximal value theorems for functions on the real line are discussed as consequences of connectedness and compactness on metric spaces. Sequences of functions are discussed, in particular uniform convergence, as well as the continuity, differentiability, integrability of the limit function.

Differentiability of functions on the real line is introduced. The mean value theorem and Taylor's theorem is discussed.

The Riemann integral in one variable is introduced. The relation between the derivative and the integral, i.e., the fundamental theorem of calculus is proved.

This course has no formal prerequisites; incoming students with a strong mathematics background are encouraged to take this class in their first semester. However, a familiarity with mathematical reasoning and proof (e.g. proof by induction or by contradiction), such as introduced in *General Mathematics and Computational Science I*, is required.

100212: Analysis II

Short Name:	Analysis II
Type:	Lecture
Credit Points:	7.5
Prerequisites:	100211
Corequisites:	100221 or 120102 or 120112 (if not already taken)
Tutorial:	Yes

Course contents This course is a continuation of *Analysis I*. Its main theme is to extend the concepts from Analysis I, in particular differentiation and integration, to functions of several variables. Taylor's theorem in several variables, the implicit function theorem and the inverse function theorem are proved. (Riemann) integration in several real variables is introduced, including the transformation formula for integrals in several variables.

100221: Linear Algebra I

Short Name:	LinAlg I
Type:	Lecture
Credit Points:	7.5
Prerequisites:	None
Corequisites:	None
Tutorial:	Yes

Course contents Together with *Analysis I*, this is one of the basic mathematics courses. It introduces vector spaces and linear maps, which play an important role throughout mathematics and its applications.

The course begins by introducing the concept of a vector space over an arbitrary field (for example, the real or complex numbers) and the concept of linear independence, leading to the notion of "dimension". We proceed to define linear maps between vector spaces and discuss properties such as nullity and rank. Linear maps can be represented by matrices and we show

how matrices can be used to compute ranks and kernels of linear maps or to solve linear systems of equations.

In order to study some geometric problems and talk about lengths and angles, we introduce an additional structure called the inner or scalar product on real vector spaces. Properties of Euclidean vector spaces and orthogonal maps are treated, including the Cauchy-Schwarz inequality, Gram-Schmidt orthonormalization and orthogonal and unitary groups.

An endomorphism is a linear map from a vector space to itself and is represented by a square matrix. We study the trace and determinant of endomorphisms and matrices and discuss eigenvalues and eigenvectors. We discuss the question whether a matrix is diagonalizable and state the theorem on Jordan Normal Form which provides a classification of endomorphisms.

This course has no formal prerequisites; incoming students with a strong mathematics background are encouraged to take this class in their first semester. However, a familiarity with mathematical reasoning and proof (e.g. proof by induction or by contradiction), such as introduced in *General Mathematics and Computational Science I*, is required.

100222: Linear Algebra II

Short Name:	LinAlg II
Type:	Lecture
Credit Points:	7.5
Prerequisites:	100221
Corequisites:	None
Tutorial:	Yes

Course contents This course continues *Analysis I* and complements *Analysis II*. It continues with the classification of matrices and introduces elements of tensor algebra.

In the first part, we continue the discussion of endomorphisms, discussing the Cayley-Hamilton Theorem and minimal polynomials, and giving some versions of Jordan normal form over non-algebraically closed fields.

The second part of the course deals with dual spaces and quadratic, symmetric and skew-symmetric forms. We introduce the dual vector space and dual linear maps and their relation with bilinear forms. Classifications are given of symmetric and skew-symmetric real bilinear forms and of Hermitian and skew-Hermitian forms over the complex numbers.

The last part is concerned with tensors — objects that play an important role in many branches of physics and mathematics. The tensor product of two (or more) vector spaces is introduced and properties are discussed including the universal property, contraction of tensors, ‘outer’ and ‘inner’ products, and the relationship between linear maps and tensors.

100291: Perspectives of Mathematics I

Short Name:	Perspectives I
Type:	Lecture
Credit Points:	5
Prerequisites:	None
Corequisites:	100211 or 100221
Tutorial:	No

Course contents This course is an overview and an introduction to selected topics from different areas of mathematics which are not usually covered in introductory classes. The goal is to develop an understanding for interesting mathematical questions beyond the standard first and second year curriculum. Rather than proving all results in full detail, we visit several different areas of mathematics, thus illustrating the breadth and beauty of mathematics.

Topics vary; past instances of this course have included combinatorial game theory, hyperbolic and spherical geometries, manifolds, Fourier analysis and wavelets, the Banach-Tarski-paradox, dynamical systems and chaos, and others.

For students in Mathematics and ACM, this course is designed to be taken at the same time as *Analysis I* and/or *Analysis I*. It is open to anyone with interest and some experience in mathematics (for others, *General Mathematics and Computational Science I* is recommended). Instead of a final exam, students write a term paper on a mathematical subject of their choice and present it to the class.

100292: Perspectives of Mathematics II

Short Name:	Perspectives II
Type:	Lecture
Credit Points:	5
Prerequisites:	100211 or 100221
Corequisites:	None
Tutorial:	No

Course contents The spring semester instance of Perspectives of Mathematics is usually, but not always, independent of the fall semester. Format and goals of this course are as for *Perspectives of Mathematics I*.

5.4 Third year Mathematics

110341: Numerical Analysis

Short Name:	NumAnal
Type:	Lecture
Credit Points:	7.5
Prerequisites:	100212
Corequisites:	None
Tutorial:	Yes

Course contents This course an advanced introduction to Numerical Analysis. It complements *ESM 4A – Numerical Methods*, placing emphasis, on the one hand, on the analysis of numerical schemes, on the other hand, focusing on problems faced in large-scale computations. Topics include sparse matrix linear algebra, large scale and/or stiff systems of ordinary differential equations, and a first introduction to methods for partial differential equations.

110361: Mathematical Modeling in Biomedical Applications

Short Name: MathMod BioMed
Type: Lecture
Credit Points: 7.5
Prerequisites: 100212
Corequisites: None
Tutorial: Yes

Course contents The course discusses the area of mathematical modeling in biomedical applications. It includes an introduction into the basic principles of mathematical modeling, and it covers a variety of models for growth and treatment of cancer with increasing complexity ranging from simple ordinary differential equations to more complicated free boundary problems and partial differential equations. Further models for the description of physiology in the human body like blood flow and breathing are briefly touched as well.

100312: Introductory Complex Analysis

Short Name: ComplexAnal
Type: Lecture
Credit Points: 7.5
Prerequisites: 100212 and 100221
Corequisites: None
Tutorial: Yes

Course contents This course introduces the theory of functions of one complex variable. It centers around the notion of complex differentiability and its various equivalent characterizations. Unlike differentiability for real functions, complex differentiability is a very strong property; for example it implies that the function is differentiable infinitely often and that it is represented by its Taylor series in a neighborhood of every point in its domain of definition. This results in a very nice and elegant theory that is used in many areas of mathematics.

Topics include holomorphic functions, Cauchy integral theorem and formula, Liouville's theorem, fundamental theorem of algebra, isolated singularities and Laurent series, analytic continuation and monodromy theorem, residue theorem, normal families and Montel's theorem, and the Riemann mapping theorem.

Possible further topics are elliptic and modular functions, the Riemann zeta function, introduction to Riemann surfaces.

100313: Real Analysis

Short Name: RealAna
Type: Lecture
Credit Points: 7.5
Prerequisites: 100212
Corequisites: None
Tutorial: Yes

Course contents Real Analysis is one of the core advanced courses in the Mathematics curriculum. It introduces measures, integration, elements from functional analysis, and the theory of function spaces. Knowledge of these topics, especially Lebesgue integration, is instrumental in many areas, in particular, for stochastic processes, partial differential equations, applied and harmonic analysis, and is a prerequisite for the graduate course in Functional Analysis.

The course is suitable for undergraduate students who have taken Analysis I/II, and Linear Algebra I; it should also be taken by incoming students of the Graduate Program in the Mathematical Sciences. Due to the central role of integration in the applied sciences, this course provides an excellent foundation for mathematically advanced students from physics and engineering.

100321: Introductory Algebra

Short Name:	IntroAlgebra
Type:	Lecture
Credit Points:	7.5
Prerequisites:	100221
Corequisites:	None
Tutorial:	Yes

Course contents This course gives an introduction to three basic types of algebraic structures: groups, commutative rings, and fields. (If time permits, a fourth one: modules.) Here is a more detailed list of topics to be covered.

Group Theory: Definitions and key examples. Cosets and Lagrange's theorem. Group homomorphisms and basic constructions including quotient groups, direct and semi-direct products. Some examples of (important) groups. Group actions and orbit-stabilizer theorem. Possibly: Sylow theorems.

(Commutative) Rings: Definitions and elementary properties. Ideals, ring homomorphisms and quotient rings. Domains, Euclidean domains, principal ideal domains and unique factorization. Polynomial rings.

Field extensions: Roots of polynomials. Irreducibiliy criteria. Finite and algebraic field extensions. Finite fields. Possibly: Splitting fields and algebraic closure. Constructions with straightedge and compass.

If time permits *Modules:* Definitions and basic constructions. Linear maps and exact sequences. Direct products and sums. Structure theory for finitely generated modules over a principal ideal domain.

100331: Introductory Number Theory

Short Name:	IntroNumTheory
Type:	Lecture
Credit Points:	7.5
Prerequisites:	100211 and 100321
Corequisites:	None
Tutorial:	Yes

Course contents This course gives a rst introduction to number theory. It starts with Elementary Number Theory, covering topics such as congruences, the Chinese Remainder Theorem, Fermat's Little Theorem and Euler's extension; these have interesting applications to cryptography (such as the famous RSA algorithm). Further topics include Gaussian integers, quadratic reciprocity, Diophantine equations, Minkowski's lattice point theorem, as well as sums of two, three, and four squares.

The course will then move on beyond elementary number theory. Depending on the interests of students and instructor, possible topics are Pell's equation and continued fractions, the Prime Number Theorem, Dirichlet's theorem about primes in arithmetic progressions, or elliptic curves.

100332: Discrete Mathematics

Short Name:	DiscMath
Type:	Lecture
Credit Points:	7.5
Prerequisites:	None
Corequisites:	None
Tutorial:	Yes

Course contents This course is open to anyone with interest and some experience in mathematics (for others, *General Mathematics and Computational Science I* and/or *General Mathematics and Computational Science II* is recommended).

Discrete mathematics is a branch of mathematics that deals with discrete objects and has naturally many applications to computer science. This course introduces the basics of the subject, in particular (enumerative) combinatorics, graph theory, as well as mathematical logic.

Enumerative combinatorics includes the binomial and multinomial coefficients, the pigeonhole principle, the inclusion-exclusion formula, generating functions, partitions, and Young diagrams.

Fundamental topics in graph theory include trees (spanning trees, enumeration of trees), cycles (Eulerian and Hamiltonian cycles), planar graphs (Kuratowski's theorem), colorings, and matching (perfect matchings, Hall's theorem).

In mathematical logic, among the basic topics are the Zermelo-Fraenkel axioms, as well as cardinal and ordinal numbers and their properties.

Additional topics may be chosen depending on interests of instructor and students.

100341: Introductory Topology

Short Name:	IntroTopology
Type:	Lecture
Credit Points:	7.5
Prerequisites:	100211 and 100221
Corequisites:	None
Tutorial:	Yes

Course contents This course is an introduction to some basic concepts and techniques in topology. The first part of the course builds on material from *Analysis I*, in particular the topology of metric spaces. We introduce topological spaces and continuous maps and proceed to discuss properties of spaces including connectedness, compactness and the Hausdorff property. Basic constructions such as the product and quotient of spaces are also treated.

The second part of the course deals with basic concepts of algebraic topology. We introduce the notion of homotopy, construct the fundamental group of a space and introduce the Seifert–van Kampen theorem, a key tool for computing fundamental groups. We discuss covering spaces and their relation with the fundamental group, including the construction of the universal covering space.

The course concludes with a basic treatment of homology groups and their properties, which are a fundamental tool for distinguishing topological spaces and mappings between them.

100353: Manifolds and Topology

Short Name: ManifoldsTop
Type: Lecture
Credit Points: 7.5
Prerequisites: 100212 and 100221
Corequisites: None
Tutorial: Yes

Course contents This course is an introduction to the language and some of the fundamental concepts of modern geometry. Manifolds are among the most fundamental concepts of mathematics: curves and surfaces are important special cases that have historical significance.

The course starts with introducing the notion of a manifold, followed by examples that naturally arise in various areas of mathematics. Differentiability, tangent spaces and vector fields are then defined. This will be followed by establishing the notion of integration on manifolds. We will then formulate and prove Stokes' theorem, which is the higher-dimensional generalization of the fundamental theorem of calculus. Among the further topics that are discussed in the course are: orientation, degree of a map, Lie groups and their actions. The classification of one- and two-dimensional manifolds and the Poincaré-Hopf theorem will be some of the highlights of the course.

100361: Ordinary Differential Equations and Dynamical Systems

Short Name: DynSystems
Type: Lecture
Credit Points: 7.5
Prerequisites: 100212 and 100221
Corequisites: None
Tutorial: Yes

Course contents Dynamical systems is an topic which links pure mathematics with applications in physics, biology, electrical engineering, and others. The course will furnish a systematic introduction to ordinary differential equations in one and several variables, focusing more on qualitative aspects of solutions than on explicit solution formulas in those few cases where such exist. It will be shown how simple differential equations can lead to complicated and interesting, often "chaotic" dynamical behavior, and that such arise naturally in the "real world".

We will also discuss time-discrete dynamical systems (iteration theory) with its relations and differences to differential equations.

100362: Introductory Partial Differential Equations

Short Name: Intro PDE
Type: Lecture
Credit Points: 7.5
Prerequisites: 100212
Corequisites: None
Tutorial: Yes

Course contents This course is a rigorous, but elementary introduction to the theory of partial differential equations: classification of PDEs, linear prototypes (transport equation, Poisson equation, heat equation, wave equation); functional setting, function spaces, variational methods, weak and strong solutions; first order nonlinear PDEs, introduction to conservation laws; exact solution techniques, transform methods, power series solutions, asymptotics.

This course alternates with *Partial Differential Equations* which takes a functional analytic approach to partial differential equations.

100382: Stochastic Processes

Short Name: StochProc
Type: Lecture
Credit Points: 7.5
Prerequisites: 100212
Corequisites: None
Tutorial: Yes

Course contents This course is an introduction to the theory of stochastic processes. The course will start with a brief review of probability theory including probability spaces, random variables, independence, conditional probability, and expectation.

The main part of the course is devoted to studying important classes of discrete and continuous time stochastic processes. In the discrete time case, topics include sequences of independent random variables, large deviation theory, Markov chains (in particular random walks on graphs), branching processes, and optimal stopping times. In the continuous time case, Poisson processes, Wiener processes (Brownian motion) and some related processes will be discussed.

This course alternates with *Applied Stochastic Processes*.

100383: Applied Stochastic Processes

Short Name: ApplStochProc
Type: Lecture
Credit Points: 7.5
Prerequisites: 100212
Corequisites: None
Tutorial: Yes

Course contents This course aims at an introduction to the mathematical theory of financial markets that discusses important theoretical concepts from the theory of stochastic processes developed in parallel to their application to the mathematical finance.

The applied part of this course revolves around the central question of option pricing in markets without arbitrage which will be first posed and fully solved in the case of binomial model. Interestingly enough, many of the fundamental concepts of financial mathematics such as arbitrage, martingale measure, replication and hedging will manifest themselves, even in this simple model. After discussing conditional expectation and martingales, more sophisticated models will be introduced that involve multiple assets and several trading dates. After discussing the fundamental theorem of asset pricing in the discrete case, the course will turn to continuous processes. The Wiener process, Itō integrals, basic stochastic calculus, combined with the main applied counterpart, the Black-Scholes model, will conclude the course.

This course alternates with *Stochastic Processes*.

100391: Guided Research Mathematics I

Short Name:	GR Math I
Type:	Self Study
Credit Points:	7.5
Prerequisites:	permission of instructor
Corequisites:	None
Tutorial:	No

Course contents Guided Research allows study, typically in the form of a research project, in a particular area of specialization that is not offered by regularly scheduled courses. Each participant must find a member of the faculty as a supervisor, and arrange to work with him or her in a small study group or on a one-on-one basis.

Guided research has three major components: Literature study, research project, and seminar presentation. The relative weight of each will vary according to topic area, the level of preparedness of the participant(s), and the number of students in the study group. Possible research tasks include formulating and proving a conjecture, proving a known theorem in a novel way, investigating a mathematical problem by computer experiments, or studying a problem of practical importance using mathematical methods.

Third year students in Mathematics and ACM are advised to take 1–2 semesters of Guided Research. The Guided Research report in the spring semester will typically be the Bachelor's Thesis which is a graduation requirement for every Jacobs University undergraduate. Note that the Bachelor's Thesis may also be written as part of any other course by arrangement with the respective instructor of record.

100392: Guided Research Mathematics and BSc Thesis II

Short Name:	GR Math II
Type:	Self Study
Credit Points:	7.5
Prerequisites:	permission of instructor
Corequisites:	None
Tutorial:	No

Course contents As for *Guided Research Mathematics I*.

5.5 Advanced courses

110411: Topics in Applied Analysis

Short Name: ApplAnalysis
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents The course Topics in Applied Analysis introduces to a variety of fundamental analytic tools and methods used in the theory, modeling, and numerical simulation of phenomena in the natural sciences. The course is offered with different contents in different years, the choice will depend on the instructor. Examples of areas currently covered are applied harmonic analysis and operator theory, perturbation theory and asymptotic analysis, approximation theory, and others. Students specializing in applied mathematics or applied sciences may participate in this course more than once.

100412: Topics in Complex Analysis

Short Name: CompAnalysis
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents Topics in Complex Analysis builds on the material taught in the undergraduate Complex Variables course. After a quick review of the most important results and concepts, some more advanced topics are covered. Possible subjects are Riemann Surfaces, Elliptic Functions and Modular Forms, Complex Dynamics, Geometric Complex Analysis, or Several Complex Variables. Which subjects are chosen will depend on the instructor and on the students' interests. This course may also provide an introduction to a specific area of research, leading to possible PhD thesis projects.

Due to the varying content, this course can be taken multiple times for credit.

100421: Algebra

Short Name: Algebra
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents Advanced topics from algebra, including groups, rings, ideals, fields, and modules, continuing the course *Introductory Algebra*.

100422: Advanced Algebra

Short Name: AdvAlg
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents This course develops more advanced topics in algebra beyond those from *Algebra*, including Galois theory, commutative algebra and its relation to algebraic geometry, as well as elements of noncommutative algebra.

100423: Algebraic Geometry

Short Name: AlgGeometry
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents Algebraic geometry is the study of geometry using algebraic tools: the geometric objects are the common roots of a set of polynomials in several variables. Many geometric properties can be studied in terms of algebraic properties of these polynomials, using the powerful machinery of algebra to study geometry.

Basic concepts from *Algebra* and *Introductory Algebra* are used in this course. Among the studied subjects are affine and projetive varieties, schemes, curves, and cohomology.

100442: Algebraic Topology

Short Name: AlgebrTopology
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents This course is mostly concerned with the comprehensive treatment of the fundamental ideas of singular homology/cohomology theory and duality. The knowledge of fundamental concepts of algebra as well as of general topology is assumed (at a level of *Introductory Topology* and *Introductory Algebra*.

The first part studies the definition of homology and the properties that lead to the axiomatic characterization of homology theory. Then further algebraic concepts such as cohomology and the multiplicative structure in cohomology are introduced. In the last section the duality between homology and cohomology of manifolds is studied and few basic elements of obstruction theory are discussed.

The graduate algebraic topology course gives a solid introduction to fundamental ideas and results that are used nowadays in most areas of pure mathematics and theoretical physics.

100451: Differential Geometry

Short Name: DiffGeom
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents Differential geometry is the study of differentiable manifolds. Assuming basic concepts such as manifolds, differential forms, and Stokes' theorem, the focus in this course is on Riemannian geometry: the study of curved spaces which is at the heart of much current mathematics as well as mathematical physics (for example, General Relativity).

100452: Lie Groups and Lie Algebras

Short Name: LieGroups
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents A Lie group is a group with a differentiable structure, the tangent space at the identity element of a Lie group is its Lie algebra. Lie groups and Lie algebras are indispensable in many areas of mathematics and physics. As a mathematical subject on its own, Lie theory has led to many beautiful results, such as the famous classification of semisimple Lie algebras. In physics, Lie groups and their representations are essential to the theory of elementary particles and its current developments. Due to the close correspondence of physical phenomena and abstract mathematical structures, the theory of Lie groups has become a showcase of mathematical physics.

The course presents fundamental concepts, methods and results of Lie theory and representation theory. It covers the relation between Lie groups and Lie algebras, structure theory of Lie algebras, classification of semisimple Lie algebras, finite-dimensional representations of Lie algebras, and tensor representations and their irreducible decompositions.

A solid background in multivariable real analysis and linear algebra is presumed. Familiarity with some basic algebra and group theory will also be helpful. No prior knowledge of differential geometry is necessary.

100453: Modern Geometry

Short Name: ModernGeometry
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents The course serves as an introduction, at the advanced level, to the basic concepts of modern geometry.

The following concepts, known from the 300-level courses, should be briefly reviewed: concept of a manifold, the simplest examples of manifolds, and the concept of homotopy.

The core of the course will consist of explaining material related to the following topics: Lie groups, homogeneous spaces, symmetric spaces, fiber bundles, vector bundles, Morse theory, differential topology of mappings and submanifolds.

This material will provide a solid background for the 400-level courses, *Differential Geometry* and *Algebraic Topology*.

100461: Dynamical Systems

Short Name: DynSystems
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents Dynamical systems is the study of the long-term behavior of anything in motion. The classical motivating topic is the stability of the solar system or, more recently, the study of weather prediction.

One theme in the course is the study of the underlying questions and difficulties in terms of model equations that are much simpler, often 1-, 2-, or at most 3-dimensional, but yet show rich and interesting dynamical features. A fundamental tool is to describe the dynamics of flows in terms of iterated maps of lower dimension, which are of great interest in their own right. Among the topics covered are circle homeomorphisms and endomorphisms, including rotation numbers, the quadratic family, toral automorphisms, horseshoes and the solenoid, the Lorenz systems, symbolic dynamics and shifts, and Sharkovski's theorem.

A second topic are ways to describe and quantify how complicated dynamical systems are: recurrence, topological transitivity and periodic orbits, mixing dynamics, topological and metric entropy, Lyapunov exponents, ergodicity and Birkhoff's theorem, and more.

Finally, there will be a discussion of general hyperbolic dynamics, including the stable/unstable manifold theorem and the shadowing lemma (not necessarily with detailed proofs in full generality).

100471: Functional Analysis

Short Name: FunctAnalysis
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents This course assumes basic knowledge of measure and integration theory, and of classical Banach and Hilbert spaces of measurable functions. Functional Analysis focuses on the description, analysis, and representation of linear functionals and operators defined on general topological vector spaces, most prominently on abstract Banach and Hilbert spaces.

Even though abstract in nature, the tools of Functional Analysis play a central role in applied mathematics, e.g., in partial differential equations. To illustrate this strength of Functional Analysis is one of the goals of this course.

100472: Partial Differential Equations

Short Name: PDE
Type: Lecture
Credit Points: 7.5
Prerequisites: 100313
Corequisites: None
Tutorial: No

Course contents The course is an introduction to the theory of partial differential equations in a Sobolev space setting. Topics include Sobolev spaces, second order elliptic equations, parabolic equations, semi-groups, and a selection of nonlinear problems.

This course differs from the approach taken in *Introductory Partial Differential Equations* which focuses on solutions in classical function spaces via Greens functions. It may therefore be taken by students who have attended *Introductory Partial Differential Equations*, but we will again start from basic principles so that *Introductory Partial Differential Equations* is not a prerequisite.